Turbulent Flow Ballistics Facility for Particle Momentum Transfer Studies

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This paper describes the calibration and use of an experimental installation which was specifically designed for the quantitative measurement of the drag coefficient of single particles moving with a turbulent fluid. The installation consisted of an 8-in. I.D. wind tunnel with an 18-ft. test section, preceded by an inlet grid arrangement designed to provide a flow with a symmetrical mean velocity profile and a flat central core having a nonperiodic spectrum of turbulence. The particles were launched at controlled entrance velocities into the core in a nonrotating manner by means of a particle accelerator, and axial motion was obtained. A hot-wire anemometer was used to measure the flow turbulence parameters at two Reynolds numbers by means of probes inserted at various distances above the inlet grid section. These measurements included the radial and axial intensities, the axial turbulent energy spectrum, and the macro and microscales. An accurate record of the particle velocity history was provided by a new radiotracer velocity measurement technique. This information was then processed by an IBM-650 program which supplied the drag coefficient as well as the instantaneous particle velocity, acceleration, rate of change of acceleration, Reynolds number, and relative turbulence intensity from the time-distance data. The experimental facility was found to be well suited for general studies of momentum transfer phenomena in particulate systems at high relative intensities.

Analysis of the information available on momentum transfer in solids-gas flow systems (*1*) has strongly indicated the need for quantitative measurements of the drag coefficient of particles moving with a turbulent fluid. The experimental approach to such measurements is admittedly difficult, since it requires precise determination of the particle velocity history in addition to an accurate knowledge of the fluid turbulence parameter throughout the particle trajectory. In view of these difficulties the lack of fundamental data in the published literature is not too surprising. In the following a turbulent flow ballistics wind tunnel is described with a radiotracer particle velocity method which was developed to allow precise measurements of the momentum transfer.

DESIGN AND CALIBRATION OF A TURBULENT FLOW, PARTICLE BALLISTICS WIND TUNNEL

Four experimental requirements must be fulfilled to allow the measurement of the fluid drag force on a particle moving concurrently with a gas stream: the particle velocity must be measured with sufficient accuracy to allow a double differentiation of the time-distance data; the trajectory must be sufficiently long to allow the drag forces to cause velocity alterations which are significant, relative to the possible experimental error; the mean velocity and turbulence characteristics of the fluid with which the particle comes into contact must be specified throughout the entire trajectory; and the particle must be introduced in a nonrotating manner and with a minimum disturbance to the fluid flow. In addition it is convenient to be able to adjust the particle entrance velocity so that the particles can be fired at both below and above their equilibrium velocity.

An unorthodox tunnel design was developed and adopted to meet these requirements as well as others to suit future turbulence researches which are anticipated. A schematic diagram of the wind tunnel is shown in Figure 1.

Apparatus

Particle Accelerator. A particle accelerator located at the tunnel entrance served to launch the single spheres at controlled entrance velocities in a manner which would not impart any rotation to them. As shown in Figure 2 the test sphere rested in the conical depression of a polyethylene sabot or plug which was accurately fitted to travel along the length of a 2.8-ft. vertical gun barrel. The sabot was accelerated up the barrel by a blast of compressed air from a reservoir, and it was impulsively brought to rest by an annular arresting plate located at the end of the barrel. Since the tunnel was operated under a slight vacuum, there being approximately 7 cm. Hg pressure drop across the grid section, the loading of the particle onto the sabot required an air-locking arrangement.

Inlet Grid Section. The inlet grid system must perform three functions: It must impart a symmetrical mean velocity profile with a flat central core to the flow entering the test section, the turbulence parameters of the flow must not be too far removed from those which occur in fully developed duct flow, and a nonperiodic spectrum must be quickly established.

The conventional method of establishing the desired mean velocity profile is to draw the air in through a seven-to-one contraction which allows the boundary layer to build up on the duct surface. For an 8-in. duct Mickelsen (*2*) found that this required more than 25 ft. of inlet section, and in addition the inlet must be far removed from any ambient obstructions. Single grids do not exert a sufficient straightening effect (*3*), and a considerable downstream distance is required before the spectrum becomes random.

The system developed consisted of four ⅛ in. thick aluminum plates with sharp accurately machined 1-in. circular orifices drilled and spaced at the apices of 2½-in. equilateral triangles. A hole placed at the center of each plate accommodated the gun barrel, and the grids were separated from each other by a series of stacked rings each one of which had a 16-in. O.D. The inner diameters of the rings were progressively reduced from 12 in. at the bottom to 8 in. at the top. The grids were carefully assembled so that the holes of each grid were exactly vertically superimposed on those of the others.

The air being drawn into the column tended to form vertical jets symmetrically placed about the barrel and tunnel axis. Laurence and Benninghoff (*4*), who worked with multiple interfering jets, found that the acoustic peaks of energy which would appear and be sustained with a single jet would rapidly be transferred to adjacent frequencies when the jet found itself in the company of similar jets placed adjacent to it. The phenomenon is somewhat similar to the breakdown of periodic fluctuations by free-stream vorticity noted in the work of Bennett and Lee (*5*). Single orifice grids can produce jets which refuse to merge if the ratio of solid-to-open area of the grid lies within a critical range (*6*), and this could produce low-frequency pulsations throughout the tunnel. Some variations in grid performance can occur if the edges in contact with the flow are rounded or are subject to changes in surface roughness (*7*), and this was avoided by the use of the sharp-edged orifices.

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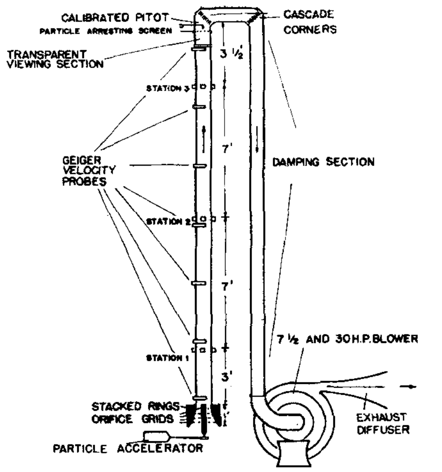

Fig. 1. Turbulent flow ballistics wind tunnel.

Tunnel Test Section. The tunnel test section consisted of an 8-in. I.D., 18 ft. long polyvinyl-chloride plastic circular duct which was mounted vertically on a vibration pad in contact with the uppermost stacked ring of the inlet section. The duct had an extremely smooth internal surface and was carefully checked for constant internal circularity.

Six 1-in. circular ports were cut in the duct wall at three levels, 3, 10, and 17 ft. downstream from the grid section. Plugs of the same material and radius of curvature as the pipe were accurately machined to fit the ports exactly.

A flanged transparent lucite section, 8-in. I.D. and approximately 1 ft. in length, was fitted into the top of the test section with a stainless steel particle-arresting screen across its upper end, and a calibrated pitot-static tube immediately above it.

Two 90-deg. corners, containing cascaded turning blades, connected the test section to the 8-in. I.D., 20 ft. long down-comer. Measurements taken 3 ft. upstream from the elbow showed no velocity-profile distortion or flow pulsation due to the cascade corners, which were constructed according to the design prescribed by Dimmock (*8*). The downcomer acted as a damping section for the possible oscillations introduced by the downstream blower section, and the tunnel wall vibration, as studied with a hot-wire anemometer fixed to the tunnel with the wire vibrating in the stagnant room air, showed the vibration amplitude to be negligible.

The blower had a maximum rated capacity of 1,500 cu. ft./min. at a gauge pressure of 50 in. of water while revolving at 3,450 rev./min. and discharged the air to the atmosphere by means of a four-to-one exhaust diffuser.

Instrumentation

Mean Velocity Measurement Equipment. Wedge probes designed and calibrated by Stachiewicz (*9*) were employed initially to establish the direction and velocity of the mean flow. Two accurate copies of the standard pitot-static tube (*10*), for which no turbulence correction is necessary (*11*), were subsequently used in the velocity-profile measurements, and their performance checked against the calibration of the wedge probes. A direct reading boundary-layer micromanometer, constructed following the specifications of the instrument developed by Smith and Murphy (*12*), was used for these measurements. It had a reliable sensitivity of 10^{-4} in. of manometer fluid, with a minimum of hysteresis and meniscus distortion properties.

Turbulence Measuring Equipment. A constant current hot-wire anemometer was used for the measurement of the turbulence parameters of the flow (*13*). Its amplifier system was fitted with a continuously variable R-C compensation with a time-constant variation between 0.23 and 30 msec. to correct for the thermal lag of the hot wire. The correctly compensated frequency range of the amplifier was from 2 to 100,000 cycles/sec. The correct setting of the compensation network was determined with the help of a square-wave calibrator used in conjunction with a cathode ray oscilloscope.

The anemometer output was fed to a random signal true root-mean-square voltmeter incorporating a thermal readout with an integrating time constant of 16 sec. Its frequency response was flat (± 0.2 decibel) for a full range of from 2 cycles/sec. to 250 kc. The accuracy in the meter scale, attenuator, and calibration was ± ½%.

A sum-difference unit was incorporated into the hot-wire anemometer facility for the correlation measurements needed in the Eulerian scale and transverse intensity evaluations. Also part of the assembly was a spectrum analyzer with a band width of 4 cycles/sec. The fidelity of its output was reestablished by a series of concentrated test-frequency trials.

Probes with Tungsten hot-wire filaments, 0.00035-in. in diameter, were used for all the measurements. Single-wire commercial probes were used, but for the X-array type a moveable head, two-wire probe was constructed in the laboratory which allowed both the separation distance and the relative angle between the two wires to be varied with minimum interwire interference, as checked with single-wire readings.

Flow Calibration

Mean Velocity Determination. Velocity profiles were obtained at the 3-, 10-, and 17-ft. levels above the grid section, referred to as *stations 1, 2, and 3,* respectively. Each station had two parts at 90 deg. to each other, allowing the flow to be traversed in two mutually perpendicular directions.

Figure 3 shows the mean velocity profiles given as the ratio of the measured velocity U to the velocity measured at the axis U_c; these profiles are flatter than those given by Sandborn (*14*) for fully developed flow. With very few exceptions the ratio was equal to 1.00 for the 2-in. central core. Particularly interesting is the result for the 3-ft. level, which shows no depression due to the presence of the barrel of the particle accelerator, located upstream of the section.

Turbulence Intensities. The turbulence intensity $\sqrt{\overline{u'^2}}/U_c$ was taken at seven levels above the grids and decreased gradually with downstream distance from a value of 8.1% 1½ ft. above the grids to 3.7% at the top of the test section. The intensity in the inlet vicinity was lower than that which would be obtained with con-

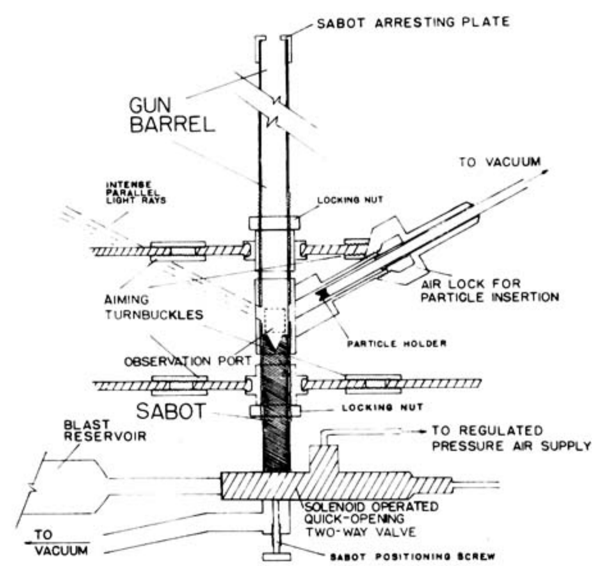

Fig. 2. Diagram of accelerator.

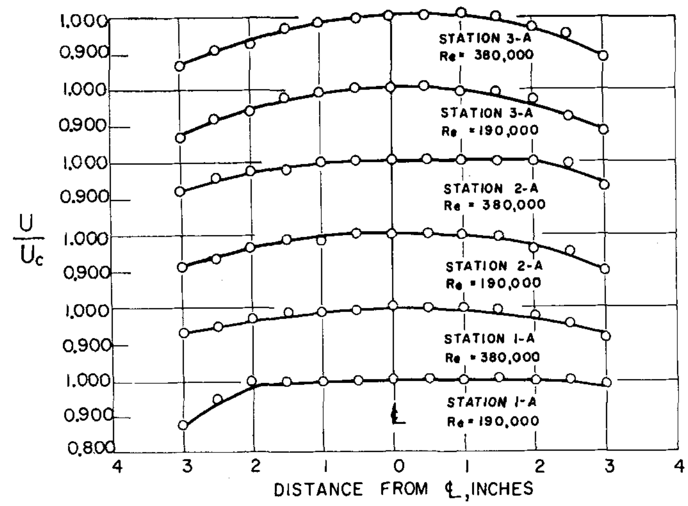

Fig. 3. Mean velocity profiles.

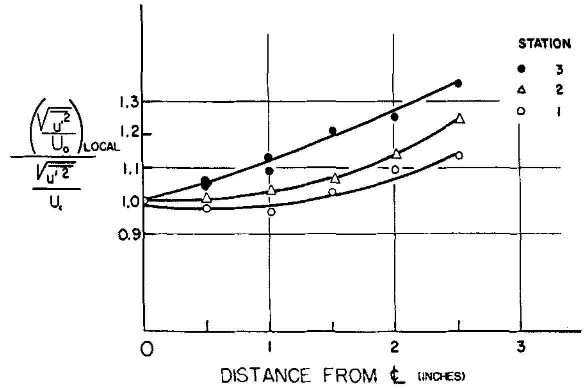

Fig. 4. Radial intensity profiles.

ventional grids or jets. Laurence and Benninghoff (*4*) also found a marked reduction in intensity and scale in the nozzle region for multiple interfering jets. The values were corrected for finite wire length according to the methods given by Frenkiel (*15*).

Radial $\sqrt{u'^2}/U_c$ traverses show profiles which are flatter than those obtained in fully developed pipe flow (see Figure 4), and this coincides with the flatter mean-velocity profiles obtained. The measurements with an X-wire array were found not to correspond with single-wire readings. A single-wire method, incorporating an alignment calibration, suggested to the authors by Newman (*16*), was finally employed, and the ratio $\sqrt{u'^2}/\sqrt{v'^2}$ was found to vary from 1.23 to 1.07 at a Reynolds number of 380,000. The flat mean-velocity profiles suggest that the ratio should approach unity.

Spectrum Measurements. The $\sqrt{u'^2}$ energy spectrum was evaluated at the three stations and at Reynolds numbers of 190,000 and 380,000. The spectral density is obtained by measuring the turbulent energy contained in a specific narrow-band width for a given intensity component and then comparing it with the total energy associated with that intensity component.

The relatively smooth curves formed by the data, as illustrated by Figure 5, show that the fluctuation energies are randomly distributed, and a very slow scanning of the frequency range failed to detect any acoustic peaks. The shape of the curves is similar for all regions, and they correspond to those usually obtained in properly developed pipe flow.

The linear portions of the curves are interesting in that their slopes come close to the $-5/3$ value predicted by Kolmogoroff (*17*) for the equilibrium range of the energy spectrum in an isotropic field. Here no attempt was made to refine the data further at the high frequencies by making corrections for the finite wire lengths used, since this would be over-shadowed by the uncertainties inherent in the measuring system. In general it is extremely difficult to obtain meaningful results at the high end of the frequency range at elevated Reynolds numbers.

Scale Measurements. The correlation functions R_y between values of u' at the pipe axis and at points separated by progressively larger radial distances from the axis were measured with the movable head scale probe, which allowed the separation distance to be adjusted to less than 0.005 in. Measurements in excess of 1-in. separation were taken with two single-wire probes entering the tunnel at two positions, but at the same tunnel level. The results obtained with the single-wire probe overlap those taken with the two-wire probe system. Intensity readings were taken with a single wire of the moveable head probe to confirm that there was no probe-holder interference. The Eulerian macroscale $L_y = \int_0^\infty R_y dy$ obtained from the area contained by the curves was found to be 0.41, 0.41, and 0.34 in. at distances of 3, 10, and 17 ft. above the grid section, respectively.

The microscale λ_y can be roughly estimated by fitting a parabola $y^2/\lambda^2_y = (1 - R_y)$ to the data, since

$$1/\lambda^2_y = \lim_{y \to 0} \left(\frac{1 - R_y}{y^2} \right) \qquad (1)$$

and values of 0.025, 0.017, and 0.015 in. were obtained for the three stations, respectively. The microscale λ_x was estimated with a technique suggested by Liepmann et al. (*18*). Oscillograph traces of the u' fluctuations were recorded photographically, and the zeros were counted for a given time interval. The microscale λ_x was then evaluated from the approximate relationship:

$$1/\lambda_x = \pi N_o / U_c \qquad (2)$$

A comparison of results obtained by this method and others was given by Laufer (*19*) which indicated that it offered a good approximation. The microscale lengths for λ_x were found to be 0.0235, 0.0245 and 0.0234 in. for the three stations respectively.

In isotropic turbulence the ratio λ_x/λ_y is theoretically predicted to have a value of 1.44. The average ratio obtained in the present measurements was 1.3 which compares favourably with the theoretical value in view of the possible errors which accompany the approximate methods employed.

A RADIOTRACER METHOD FOR PARTICLE VELOCITY MEASUREMENTS IN SOLIDS-GAS SYSTEMS

Fundamental studies involving the transfer of heat, mass, or momentum from particles entrained in a moving fluid share a common difficulty in that they require an accurate knowledge of the particle velocity history. In addition to being capable of good reproducibility a suitable technique must allow for measurements to be taken simultaneously at many points in a flow field, which could be over a foot in diameter, without the introduction of any disturbance. Such a technique was developed by the authors in collaboration with others and has been described in a previous paper in detail (*20*). Its accuracy has already been demonstrated by Pasternak, who used it in his studies of the rate of mass transfer from moving particles (*21*).

In essence the method consists of following the flight of the particle under test, which has been previously tagged with a suitable gamma-emit-

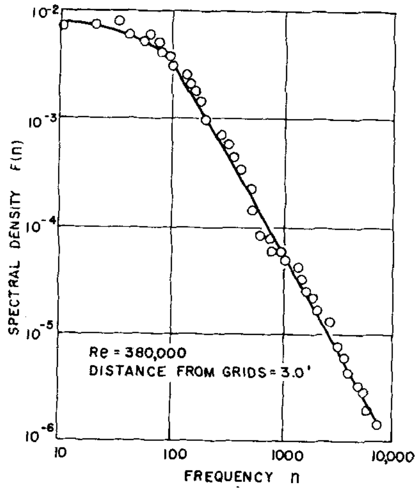

Fig. 5. Spectral density as a function of frequency.

ting nuclide, by means of Geiger probes encased in heavy lead shields with narrow slits and placed at predetermined points along the outside surface of the duct and parallel to its axis. As the particle passes each probe, it fires the Geiger circuit, producing a sharp timing pulse on the screen of a cathode-ray oscilloscope, to the vertical poles of which the probes are connected in parallel. The distance between pulses, suitably photographed, is referred to a signal produced simultaneously from a crystal calibrated pulse generator and permits the estimation of the time elapsed between probe stations to the nearest 1/10,000 sec.

The minimum surface activation level required was found to be approximately 30 mr./hr. The natural neutron-absorbing ability of spheres having a wide range of densities and sizes could not be relied upon to achieve the required activation level, with the neutron fluxes which were available. Sufficiently radioactive spheres were however prepared in the following manner. Small irregular particles of the desired material (Celite and pith were employed) were placed in a ball-making apparatus consisting simply of a circular race, 2 in. in diameter and ½ in. in depth, lined with abrasive strips of varying degrees of roughness. A tangential air jet issuing from a hypodermic needle whirled the particles along the surface of the race, the attrition process shaping them into very accurate spheres in a period generally not exceeding 3 min. The spheres were then soaked in a solution of iridium trichloride and dried under vacuum so that a fraction of a milligram of the salt remained in the interstices of the porous material. The particles were then coated with a plastic film and were subsequently restored to accurate sphericity by being rerun in the ball apparatus, operated without an abrasive strip. In this manner the radioactive tracer, when activated, was insulated from contact with surfaces which it could contaminate. The particles prepared in this manner were sealed in individual quartz tubes and irradiated in a reactor for a period of 72 hr. in a position having a flux density of 8.6 x 10^{12} neutrons/(sec.)(sq.cm.).

The accuracy and reproducibility of the method were tested initially by repeated dropping of heavy active spheres in still air. With the shielded probes it was possible to obtain sharp pulses with the probes more than 12 in. from the axis of fall.

Since graphical differentiation methods are usually a large source of error in drag coefficient studies, the time-distance results obtained in actual firings were processed by an IBM-650 program which supplied the drag coefficients as well as the particle velocity, acceleration, rate of change of acceleration, Reynolds number, and relative intensity from the readings taken. A fourth-order polynomial was used to fit the time-distance data, and the computer readout showed that the results were accurately fitted and were not upgraded by a smoothing operation.

A plot of a typical velocity history is shown in Figure 6, and the results are free from any irregularities. During a firing sequence with a given particle the pulse pattern for similar entrance velocities should be identical. Superimposition of the pulse patterns from two consecutive firings showed that the pulse peaks matched perfectly. The method was also found sensitive enough to detect departure from a strictly axial path, since it was observed that the pulse height was markedly affected by the square of the lateral distance of the particle from the Geiger tube. Only when the particle followed an axial path would the pulses for consecutive tests match in height. As an illustration a thin plastic fleck was fastened to the rear of one of the test spheres, which caused it to have a wobbling motion although it did not drift into the region of the wall. Comparison with previous pulse patterns made the nonaxial motion readily discernible. A departure from the tunnel axis parallel to the axis of the probe would unfortunately not be detected with the system used.

CONCLUSIONS

The turbulent flow particle ballistics wind tunnel was found to have the flow properties required to allow the quantitative measurement of the drag coefficients of single particles in flight through a moving fluid, and the difficulty of rapidly establishing a random flow would seem to have been overcome. The test bodies were exposed to very high relative intensities which cannot be obtained or measured in stationary or floating experiments, and the particle accelerator succeeded in confining the test body trajectory to the immediate vicinity of the tunnel axis. Possible detrimental effects due to the presence of the gun barrel and the equipment at the tunnel entrance were eliminated by the highly dissipative inlet section employed.

Because of the flow conditions produced by the inlet system the tunnel is readily adaptable to generalized turbulence studies of objects in flight which could involve heat and mass as well as momentum transfer. More immediately the launching method employed is being modified to fire nonspherical bodies as well as assemblages of particles. The very small residence time in the tunnel has the advantage of minimizing the difficulties normally encountered with particles which would tend to develop a nonaxial motion.

In addition to the accuracy and continual tracking provided by the radiotracer technique it offers further advantages over former methods by enabling measurements which could not previously be made. These include particulate movement in bends, nontransparent systems, and wide-diameter vessels.

Two undesirable attributes of the installation should be mentioned. The measured noise level in the vicinity of the grids is close to the amount needed to cause permanent ear damage. The authors used ear plugs when the tunnel was in operation, but the irritations to other personnel require the entire tunnel installation to be housed in a properly sound-proofed room. A second difficulty develops from the

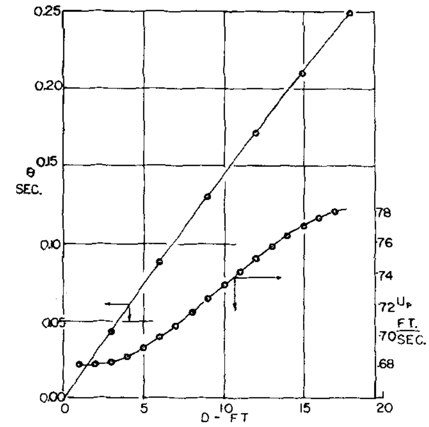

Fig. 6. Plot of particle velocity history.

need to maintain strict supervision over the handling of the radioactive materials. The added precautions necessary in the tunnel operation are tolerable in view of the measurements which are made possible.

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NOTATION

Any consistent set of units may be employed. Those listed are merely illustrative.

D = distance above grids, ft.
$F(n)$ = spectral density
L_y = Eulerian macroscale, ft.
n = frequency
N_o = average number of zero values of u' per second
R_y = Eulerian correlation coefficient, dimensionless
u' = fluctuating component of fluid velocity in axial direction, ft./sec.
U = mean fluid velocity measured at any radial distance from duct axis, ft./sec.
U_c = mean fluid velocity measured at duct axis, ft./sec.
U_p = absolute particle velocity, ft./sec.
v' = fluctuating component of fluid velocity in radial direction ft./sec.
x = longitudinal distance along duct axis, ft.
y = radial distance from duct axis, ft.
λ_x = Eulerian microscale in x direction, ft.
λ_y = Eulerian microscale in y direction, ft.
θ = time, sec.

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Adsorption Kinetics of a Nonflow System with Nonlinear Equilibrium Relationship

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The kinetics of adsorption in a nonflow system with nonlinear equilibrium relationship and negligible liquid-phase resistance has been studied, and the numerical solutions corresponding to different initial conditions are presented. The concentration ratio is found to be a function of two dimensionless groups. Consideration is also given to the problem of convergence in connection with the numerical calculation.

The study of the problem on the adsorption rate in a nonflow system is an old one. Analytical solutions for cases with linear adsorption isotherms have been obtained by Berthier (*1*), Carman and Haul (*2*), and Grober (*4*), respectively. Edeskuty and Amundson (*3*) considered the effect of intraparticle diffusion and assumed that the concentration of the liquid solution in the interior of the particles is different from that in the exterior liquid, with a linear equilibrium relationship. For cases where a nonlinear equilibrium relationship is observed, it has been found highly unlikely to obtain the solution analytically.

The object of this investigation is to treat this problem by a numerical method and present results which can be used in industrial design work. Some of the treatments involved in this work parallel closely those of a previous paper by this author (*6*) in which the adsorption kinetics for cases with nonlinear equilibrium relationship in a flow system were studied.

MATHEMATICAL DEVELOPMENT

A simple model for the adsorption process is postulated as follows. A vessel containing a definite amount of solution of known concentration is brought into contact with a known quantity of solute-free solid particles. These particles are spherical and their sizes identical. Rigorous agitation is provided which eliminates completely the film resistance of mass transfer in the liquid phase. In other words this is tantamount to assuming that solid-phase diffusion is the controlling mechanism. Furthermore it is assumed that the particles are quasihomogeneous in structure and equilibrium exists everywhere at the surface of the particle. Adsorption will take place and continue until the concentration through-